

Integrals

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Integrals

$$\begin{aligned} \uparrow F(x) &= \sin x \\ F'(x) &= \cos x \end{aligned}$$

Anti Derivative (Integration)

$$\begin{aligned} F(x) &= \sin x \\ F'(x) &= \cos x \end{aligned}$$

$$\int \cos x \, dx = \sin x + C$$

$$F(x) = \sin x + 2$$

$$F'(x) = \cos x + 0$$

$$\int \cos x \, dx = \sin x + C$$

Integrals

Derivatives	Integrals(Anti Derivatives)
$\frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = x^n$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
$\frac{d}{dx} (\sin x) = \cos x$	$\int \cos x dx = \sin x + c$
$\frac{d}{dx} (\cos x) = -\sin x$	$\int \sin x dx = -\cos x + c$
$\frac{d}{dx} (\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + c$
$\frac{d}{dx} (-\cot x) = \operatorname{Cosec}^2 x$	$\int \operatorname{Cosec}^2 x dx = -\cot x + c$
$\frac{d}{dx} (-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$
$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

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	Derivatives	Integrals(Anti Derivatives)
	$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + c$
	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} = \tan^{-1} x + c$
	$\frac{d}{dx} e^x = e^x$	$\int e^x dx = e^x + c$
	$\frac{d}{dx} \log x = \frac{1}{x}$	$\int \frac{1}{x} dx = \log x + c$
	$\frac{d}{dx} \left(\frac{a^x}{\log a} \right) = a^x$	$\int a^x dx = \frac{a^x}{\log a} + c$

Integrals



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Evaluate the following:-

$$\int \frac{e^{\log_e \sqrt{x}}}{x} dx$$

$$I = \int \frac{e^{\log_e \sqrt{x}}}{x} dx$$

$$= \int \frac{\sqrt{x}}{x} dx$$

$$= \int (x)^{1/2} \times x^{-1} dx$$

$$= \int x^{\frac{1}{2}-1} dx$$

$$= \int x^{-\frac{1}{2}} dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{x^{1/2}}{1/2} + C$$

$$I = 2\sqrt{x} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (2x^2 + 3)(1 - 3x) dx$$

$$I = \int (2x^2 + 3)(1 - 3x) dx$$

$$= \int (2x^2 - 6x^3 + 3 - 9x) dx$$

$$= \int 2x^2 dx - 6 \int x^3 dx + 3 \int 1 \cdot dx - 9 \int x dx$$

$$= \frac{2x^3}{3} - \frac{6x^4}{4} + 3x - 9 \cdot \frac{x^2}{2} + C$$

$$= \boxed{\frac{2}{3}x^3 - \frac{3x^4}{2} + 3x - \frac{9}{2}x^2 + C}$$

$$\int \sqrt{1 - \sin 2x} \, dx$$

$$I = \int \sqrt{1 - \sin 2x} \, dx$$

$$= \int \sqrt{(\sin^2 x + \cos^2 x - 2 \sin x \cos x)}$$

$$= \int \sqrt{(\sin x - \cos x)^2}$$

$$= \int (\sin x - \cos x) \, dx$$

$$= \int \sin x \, dx - \int \cos x \, dx$$

$$= -\cos x - \sin x + c$$

$$I = -(\sin x + \cos x) + c$$

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

$$I = \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \left[\frac{\sin^3 x}{\cancel{\sin^2 x} \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cancel{\cos^2 x}} \right] dx$$

$$= \int \frac{\sin x}{\cos x \cos x} dx + \int \frac{\cos x}{\sin x \sin x} dx$$

$$= \int \tan x \sec x dx + \int \cot x \operatorname{cosec} x dx$$

$$I = \sec x - \operatorname{cosec} x + c$$

$$\int \tan^{-1}(\operatorname{cosec} x + \cot x) dx$$

$$I = \int \tan^{-1}\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right) dx$$

$$= \int \tan^{-1}\left[\frac{\cancel{1} + 2\cos^2 \frac{x}{2} - \cancel{1}}{2\sin \frac{x}{2} \cos \frac{x}{2}}\right] dx$$

$$= \int \tan^{-1}\left[\frac{\cancel{2}\cos^2 \frac{x}{2}}{\cancel{2}\sin \frac{x}{2} \cos \frac{x}{2}}\right] dx$$

$$= \int \tan^{-1}\left[\frac{\cos \frac{x}{2}}{\sin \frac{x}{2}}\right] dx$$

$$= \int \tan^{-1}\left[\cot \frac{x}{2}\right] dx$$

$$= \int \tan^{-1}\left[\tan\left[\frac{\pi}{2} - \frac{x}{2}\right]\right] dx$$

$$= \int \left[\frac{\pi}{2} - \frac{x}{2}\right] dx$$

$$I = \frac{\pi}{2}x - \frac{x^2}{4} + c$$

$$\int (2^x + 2^{-x})^2 dx$$

$$I = \int ((2^x)^2 + (2^{-x})^2 + 2 \times 2^x \times 2^{-x}) dx$$

$$= \int \left[2^{2x} + 2^{-2x} + 2 \times \cancel{2^x} \times \frac{1}{\cancel{2^x}} \right] dx$$

$$\int a^x dx = \frac{a^x}{\log a} + C$$

$$= \int 2^{2x} dx + \int 2^{-2x} dx + 2 \int 1 \cdot dx$$

let $-2x = t$
 $-2 = \frac{dt}{dx}$

let $2x = t$
 $2 = \frac{dt}{dx}$

$$= \frac{1}{2} \int 2^t dt - \frac{1}{2} \int 2^t dt + 2x$$

$$dx = -\frac{dt}{2}$$

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2} \frac{2^t}{\log 2} - \frac{1}{2} \frac{2^t}{\log 2} + 2x + C$$

$$\int \frac{2x}{(2x+1)^2} dx$$

$$I = \int \frac{2x}{(2x+1)^2} dx$$

$$= \int \frac{(2x+1-1)}{(2x+1)^2} dx$$

$$= \int \frac{2x+1}{(2x+1)^2} dx - \int \frac{1}{(2x+1)^2} dx$$

$$= \int \frac{1}{2x+1} dx - \int (2x+1)^{-2} dx$$

$$\frac{1}{2} \int \frac{1}{t} dt - \frac{1}{2} \int v^{-2} dv$$

$$\frac{1}{2} \left[\log t - \frac{1}{2} \left(\frac{v^{-2+1}}{-2+1} \right) \right] + C$$

$$\frac{1}{2} \left[\log(2x+1) - \frac{1}{2} \frac{(2x+1)^{-1}}{-1} \right] + C$$

$$\frac{1}{2} \left[\log(2x+1) + \frac{1}{2(2x+1)} \right] + C$$

$$\text{let } 2x+1 = t$$

$$2 = \frac{dt}{dx}$$

$$\boxed{dx = \frac{dt}{2}}$$

$$2x+1 = v$$

$$2 = \frac{dv}{dx}$$

$$\boxed{dx = \frac{dv}{2}}$$

$$\int \sin 3x \cos 4x dx$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$



$$I = \frac{2}{2} \int \sin 3x \cos 4x dx$$

$$\sin(-x) = -\sin x$$

$$= \frac{1}{2} \int 2 \sin 3x \cos 4x dx$$

$$= \frac{1}{2} \int [\sin(3x+4x) + \sin(3x-4x)] dx$$

$$= \frac{1}{2} \left[\int \sin 7x dx - \int \sin x dx \right]$$

$$= \frac{1}{2} \left[-\frac{\cos 7x}{7} - \cos x \right] + C = \frac{-\cos 7x}{14} - \frac{\cos x}{2} + C$$

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$$\int \sin^3 x \cos^3 x dx$$

$$I = \int \sin^3 x \cos^3 x dx$$

$$\int \left(\frac{2}{2} \sin x \cos x \right)^3 dx$$

$$= \frac{1}{8} \int (2 \sin x \cos x)^3 dx$$

$$= \frac{1}{8} \int (\sin 2x)^3 dx$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$4 \sin^3 x = 3 \sin x - \sin 3x$$

$$\sin^3 x = \frac{3 \sin x - \sin 3x}{4}$$

$$\frac{1}{8} \times \frac{1}{4} \int (3 \sin 2x - \sin 3(2x)) dx$$

$$= \frac{1}{32} \left[3 \int \sin 2x dx - \int \sin 6x dx \right]$$

$$= \frac{1}{32} \left[-\frac{3}{2} \cos 2x + \frac{\cos 6x}{6} \right] + c$$

$$= \frac{1}{32} \left[\frac{-9 \cos 2x + \cos 6x}{2} \right] + c$$

$$\int \frac{\sin 2x + 3 \cos x}{(\sin^2 x + 3 \sin x + 5)^4} dx$$

$$I = \int \frac{\sin 2x + 3 \cos x}{(\sin^2 x + 3 \sin x + 5)^4} dx$$

$$= \int \frac{dt}{t^4} = \int t^{-4} dt = \frac{t^{-4+1}}{-4+1} + C$$

$$= \frac{t^{-3}}{-3} + C = -\frac{1}{3} \times \frac{1}{t^3} + C$$

$$I = -\frac{1}{3(\sin^2 x + 3 \sin x + 5)^3} + C$$

$$\text{Let } \sin^2 x + 3 \sin x + 5 = t$$

$$2 \sin x \cos x + 3 \cos x = \frac{dt}{dx}$$

$$(2 \sin x \cos x + 3 \cos x) dx = dt$$

$$(\sin 2x + 3 \cos x) dx = dt$$

$$\int \frac{dx}{x \cos^2(1 + \log x)}$$

$$\text{let } 1 + \log x = t$$

$$\frac{1}{x} dx = dt$$

$$I = \int \frac{dx}{x \cos^2(1 + \log x)}$$

$$= \int \frac{dt}{\cos^2 t} = \int \sec^2 t + C = \tan t + C = \tan(1 + \log x) + C$$



$$\int \frac{\sec^2(2\tan^{-1}x)}{1+x^2} dx$$

$$I = \frac{1}{2} \int \sec^2 t \, dt$$

$$= \frac{1}{2} \tan t + c$$

$$I = \frac{1}{2} \tan(2\tan^{-1}x) + c$$

$$2\tan^{-1}x = t$$

$$2 \times \frac{1}{1+x^2} = \frac{dt}{dx}$$

$$\frac{dx}{1+x^2} = \frac{dt}{2}$$

$$\int x^2 e^{x^3} \cos(2e^{x^3}) dx$$

$$I = \int x^2 e^{x^3} \cos(2e^{x^3}) dx$$

$$= \frac{1}{6} \int \cos t \, dt$$

$$= \frac{1}{6} (\sin t) + C$$

$$I = \frac{1}{6} \sin(2e^{x^3}) + C$$

let

$$2e^{x^3} = t$$

$$2e^{x^3} \frac{d}{dx} x^3 = \frac{dt}{dx}$$

$$2e^{x^3} \times 3x^2 = \frac{dt}{dx}$$

$$6x^2 e^{x^3} = \frac{dt}{dx}$$

$$\int e^{3 \log x} (x^4 + 1)^{-1} dx$$

$$\text{but } x^4 + 1 = t$$

$$4x^3 = \frac{dt}{dx}$$

$$x^3 dx = \frac{dt}{4}$$

$$I = \int e^{\log x^3} (x^4 + 1)^{-1} dx$$

$$= \int x^3 \times \frac{1}{(x^4 + 1)} dx$$

$$= \frac{1}{4} \int \frac{1}{t} + c$$

$$I = \frac{1}{4} \log |t| + c \Rightarrow \frac{1}{4} \log |x^4 + 1| + c$$

$$\int \cot x \log \sin x \, dx$$

$$\text{let } \log \sin x = t$$

$$I = \int \cot x \log \sin x \, dx$$

$$\frac{1}{\sin x} \times \cos x = \frac{dt}{dx}$$

$$I = \int t \, dt = \frac{t^2}{2} + c = \frac{(\log \sin x)^2}{2} + c$$

$$\cot x \, dx = dt$$

$$\int \sec x \log |\sec x + \tan x| dx$$

$$I = \int \sec x \log (\sec x + \tan x) dx$$

$$= \int t dt$$

$$I = \frac{t^2}{2} + c = \frac{[\log(\sec x + \tan x)]^2}{2} + c$$

$$\text{Let } \log(\sec x + \tan x) = t$$

$$\frac{1}{\sec x + \tan x} (\sec x + \tan x + \sec^2 x) = \frac{dt}{dx}$$

$$\frac{\sec x (\cancel{\tan x} + \sec x)}{(\sec x + \cancel{\tan x})} = \frac{dt}{dx}$$

$$\sec x = \frac{dt}{dx}$$

$$\sec x dx = dt$$

$$\int \frac{e^{2x}}{2+e^x} dx$$

$$I = \int \frac{e^{2x}}{2+e^x} dx$$

$$= \int \frac{e^x \cdot e^x}{2+e^x} dx$$

$$= \int \frac{t}{2+t} dt$$

$$= \int \left(\frac{t+2-2}{2+t} \right) dt$$

$$= \int \frac{t+2}{2+t} dt - \int \frac{2}{2+t} dt$$

$$= \int 1 \cdot dt - 2 \log(2+t) + c$$

$$= t - \log(2+t)^2 + c$$

$$I = e^x - \log(2+e^x)^2 + c$$

$$e^x = t$$

$$e^x = \frac{dt}{dx}$$

$$\int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx$$

$$I = \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx$$

$$= - \int \cos^2 t \, dt$$

$$= - \int \left(\frac{\cos 2t + 1}{2} \right) dt$$

$$= - \frac{1}{2} \left[\int \cos 2t \, dt + \int 1 \cdot dt \right]$$

$$= - \frac{1}{2} \left[\frac{\sin 2t}{2} + t \right] + c$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos 2x + 1 = 2 \cos^2 x$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\text{let } \frac{1}{x} = t$$

$$x^{-1} = t$$

$$-x^{-2} = \frac{dt}{dx}$$

$$-\frac{1}{x^2} = \frac{dt}{dx}$$

$$= - \frac{1}{2} \left[\frac{\sin 2x \cdot \frac{1}{x}}{2} + \frac{1}{x} \right] + c$$

$$-\frac{1}{x^2} dx = dt$$

Ans

$$\int \sin^5 x \, dx$$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$2 \sin^2 x = 1 - \cos 2x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\begin{aligned} \text{let} \\ \cos x &= t \\ -\sin x \, dx &= dt \end{aligned}$$

$$I = \int \sin^5 x \, dx$$

$$= \int (\sin^2 x)^2 \sin x \, dx$$

$$= \int (1 - \cos^2 x)^2 \sin x \, dx$$

$$= -\int (1 - t^2)^2 \, dt$$

$$= -\int (1 + t^4 - 2t^2) \, dt$$

$$= -\left[\int 1 \cdot dt + \int t^4 \, dt - 2 \int (t^2) \, dt \right]$$

$$= -\left[t + \frac{t^5}{5} - 2 \frac{t^3}{3} \right] + c$$

$$= -\left[\cos x + \frac{\cos^5 x}{5} - \frac{2}{3} \cos^3 x \right] + c$$

$$\int \tan^8 x \sec^4 x \, dx$$

$$I = \int \tan^8 x \sec^4 x \, dx$$

$$= \int \tan^8 x (\sec^2 x) (\sec^2 x) \, dx$$

$$= \int \tan^8 x (1 + \tan^2 x) \sec^2 x \, dx$$

$$= \int t^8 (1 + t^2) \, dt$$

$$= \int t^8 \, dt + \int t^8 \cdot t^2 \, dt$$

$$= \int t^8 \, dt + \int t^{10} \, dt$$

$$\tan x = t$$

$$\sec^2 x \, dx = dt$$

$$I = \frac{t^9}{9} + \frac{t^{11}}{11} + C$$

$$I = \frac{\tan^9 x}{9} + \frac{\tan^{11} x}{11} + C$$

$$\int \frac{x^3}{(x^2 + 1)^3} dx$$

$$I = \int \frac{x^3}{(x^2 + 1)^3} dx$$

$$= \int \frac{x^2 \cdot x}{(x^2 + 1)^3} dx$$

$$= \frac{1}{2} \int \frac{(t-1)}{t^3} dt$$

$$\frac{1}{2} \left[\int \frac{t}{t^3} dt - \int \frac{1}{t^3} dt \right] \text{ let } \left\{ \begin{array}{l} x^2 + 1 = t \\ 2x dx = dt \end{array} \right.$$

$$\frac{1}{2} \left[\int \frac{1}{t^2} dt - \int t^{-3} dt \right]$$

$$\frac{1}{2} \left[\int t^{-2} dt - \int t^{-3} dt \right]$$

$$\frac{1}{2} \left[\frac{t^{-1}}{-1} - \frac{t^{-2}}{-2} \right] + c$$

$$\frac{1}{2} \left[-\frac{1}{t} + \frac{1}{2} \times \frac{1}{t^2} \right] + c$$

$$\frac{1}{2} \left[-\frac{1}{(x^2 + 1)} + \frac{1}{2(x^2 + 1)^2} \right] + c \Rightarrow$$

$$x^2 + 1 = t$$

$$2x dx = dt$$

$$x dx = \frac{dt}{2}$$

$$x^2 = t - 1$$

$$\int \frac{x^2}{\sqrt{1+x}} dx$$

$$I = \int \frac{(t^2-1)^2}{*} \times 2t dt$$

$$= 2 \int (t^2-1)^2 dt$$

H.W

$$\sqrt{1+x} = t$$

Sq. Both Side

$$(1+x) = t^2 \Rightarrow x = t^2 - 1$$

$$1 = 2t \frac{dt}{dx}$$

$$dx = 2t dt$$

$$\int \frac{\cos(x+a)}{\sin(x+b)} dx$$



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$$\int \frac{dx}{\sin x + \sqrt{3} \cos x}$$



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Evaluate the following integrals:

$$\int \frac{dx}{4x^2 - 9}$$

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$$\int \frac{\cos x \, dx}{1 + 9\sin^2 x}$$



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$$\int \frac{dx}{e^x + e^{-x}} dx$$



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$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$



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$$\int \frac{dx}{x^2 + 6x + 12}$$



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$$\int \frac{3x + 1}{2x^2 - 2x + 3} dx$$

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$$\int \frac{5x - 2}{1 + 2x + 3x^2} dx$$

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$$\int x e^{3x} dx$$

$$\int f(x) g(x) dx = f(x) \int g(x) dx - \int \left(\frac{d}{dx} f(x) \cdot \int g(x) dx \right) dx$$

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$$I = \int \underset{\substack{\downarrow \\ I}}{x} \underset{\substack{\downarrow \\ II}}{e^{3x}} dx$$

$$= x \int e^{3x} dx - \int \left(\frac{d}{dx} x \int e^{3x} dx \right) dx$$

$$= \frac{x e^{3x}}{3} - \int 1 \cdot \frac{e^{3x}}{3} dx = \frac{x}{3} e^{3x} - \frac{e^{3x}}{3 \times 3} + C = \boxed{\frac{x}{3} e^{3x} - \frac{1}{9} e^{3x} + C}$$

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$$\int x \tan^{-1} x \, dx$$



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$$\int x^3 \tan^2(x^2) dx$$



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$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$$



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$$\int x(\log x)^2 dx$$



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$$\int \sqrt{x^2 - 8x + 7} \, dx$$



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Thank You!

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